DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

TEST BOOKLET NO-25017

MAIN PAPER

Time : 2:00 hrs

Total Marks: 100

STATISTICS

INSTRUCTIONS TO CANDIDATES

Read the instructions carefully before answering the questions: -

1. This Test Booklet consists of 16 (sixteen) pages and has 50 (Fifty) **items (questions)**.
2. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
3. **Please note that it is the candidate's responsibility to fill in the Roll Number and other required details carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the OMR Answer Sheet liable for rejection.**
4. Do not write anything else on the OMR Answer Sheet except the required information. Before you proceed to mark in the OMR Answer Sheet, please ensure that you have filled in the required particulars as per given instructions.
5. Use **only Black Ball Point Pen** to fill the OMR Answer Sheet.
6. **This booklet consists of Multiple Choice-based Questions.** The answers to these questions have to be marked in the **OMR Answer Sheet** provided to you.
7. Each item (question) comprises of 04 (four) responses (answers). You are required to select the response which you want to mark on the OMR Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
8. After you have completed filling in all your responses on the OMR Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***the OMR Answer Sheet*** . You are permitted to take the Test Booklet with you.
9. **Penalty for wrong answers in Multiple Choice-based Questions:**
   THERE WIL BE **PENALTY** FOR WRONG ANSWERS MARKED BY A CANDIDATE.
   (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to the question will be deducted as penalty.
   (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to the question.
   (iii) If a question is left blank. i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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MULTIPLE CHOICE BASED QUESTIONS

Instructions for Questions 1 to 50:

- Choose the correct answer for the following questions
- Each question carries 2 marks (50 x 2 = 100 marks)

1. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer, and $(1-p)$ be the probability that she guesses. Assuming that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. The conditional probability that a student knew the answer to a question given that she answered it correctly, is

(A) $\frac{mp}{1+mp}$

(B) $\frac{p}{m+(1-p)m^2}$

(C) $\frac{mp}{1+(m-1)p}$

(D) $\frac{p^2}{1+mp}$

2. Four fair coins are flipped. If the outcomes are assumed independent, then the probability of obtaining two heads and two tails is

(A) $\frac{3}{8}$

(B) $\frac{1}{16}$

(C) $\frac{2}{18}$

(D) None of above

3. Let X be a Geometric (0.4) random variable. Then $P(X=5|X\geq 2)$ is approximately given by

(A) 0.09

(B) 0.05

(C) 0.50

(D) 0.10

4. If X~ Poisson (0.5), then E[(X+1)!] is

(A) $e^{-1/2}$

(B) $\frac{1}{4}e^{-1/2}$

(C) $\frac{1}{4}e^{1/2}$

(D) $4e^{-1/2}$

5. If X is uniformly distributed over (0,10), then the probability that X>7 is:

(A) $\frac{7}{10}$

(B) $\frac{5}{10}$

(C) $\frac{3}{10}$

(D) $\frac{1}{10}$

6. The moment generating function of a normal variate X~N (μ, σ^2) is given by:

(A) $e^{\mu t/\sigma}$

(B) $e^{\mu t+\left(\frac{t^2}{2\sigma^2}\right)}$

(C) $e^{\mu t/2\sigma^2}$

(D) $e^{\mu t+(t^2\sigma^2/2)}$

7. Which of the following distributions has its mean greater than its variance?

(A) Binomial
(B) Poisson
(C) Exponential
(D) None of the above

8. The joint probability density function of X and Y is given by:

$$f(x,y)=\begin{cases} 6xy(2-x-y); & 0<x<1,\ 0<y<1 \\ 0 & ;\text{otherwise} \end{cases}$$

Then, the conditional expectation E[X|Y=y] is

(A) $\frac{15-7y}{8}$

B) $\frac{7y}{8} - 5$

(C) $\frac{7y}{8-6y}$

(D) $\frac{5-4y}{8-6y}$

9. Hari reads either an English poetry book or a regional language poetry book. If the number of misprints in the English book follows a Poisson distribution with mean 2 and the number of misprints in the regional book follows a Poisson distribution with mean 5, then assuming that Hari is equally likely to choose between either of the books, the expected number of misprints he encounters is:

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{11}{2}$

10. Let X_1 and X_2 be two *i.i.d.* discrete random variables with the following *pmf*:

$$p(x) = \begin{cases} \left(\frac{1}{2}\right)^x & ; x = 1,2, \ldots \\ 0 & ; \text{otherwise} \end{cases}$$

Then, the Probability (min $(X_1, X_2) < 3$) is:

(A) $\frac{5}{6}$

(B) $\frac{3}{6}$

(C) $\frac{15}{16}$

(D) $\frac{13}{16}$

11. A miner is trapped in a mine containing 03 doors. The first door leads to a tunnel that takes him to safety in after two hours of travel. Second door leads to a tunnel that returns him to the mine after three hours of travel. Third door leads to a tunnel that returns him to the mine after five hours of travel. Assuming that the miner is at all times equally likely to choose any one of the doors, the expected length of time until the miner reaches safety is:

(A) 08 hours

(B) 15 hours

(C) 12 hours

(D) 10 hours

12. Which of the following statements are correct?

(i) Mode of F-distribution is always less than unity.
(ii) Sum of independent exponential variates is a Gamma variate.

(A) Only (i)
(B) Only (ii)
(C) Both (i) and (ii)
(D) None

13. Which of the following statements are incorrect?

(i) Limiting distribution of a chi-square variate with n degrees of freedom is a normal distribution with mean $2n$ and variance n.

(ii) Square of standard normal variate follows a chi-square distribution with 1 degree of freedom

(A) Only (i)
(B) Only (ii)
(C) Both (i) and (ii)
(D) None

14. If $X_1,..., X_n$ is a random sample from a Normal population $N(\mu,1)$, then an unbiased estimate of $(1+\mu^2)$ is:

(A) $1+\sum_{i=1}^{n} x_i^2$

(B) $n+\sum_{i=1}^{n} x_i^2$

(C) $\sum_{i=1}^{n} x_i^2 - 1$

(D) $\sum_{i=1}^{n} x_i^2$

15. Consider the following statements:

(i) If T is sufficient for θ, then any one-to-one function of T is also sufficient for θ.
(ii) Every function of a sufficient statistic is itself sufficient.

Which of the following claims are true?

(A) Both statements (i) and (ii) are correct, and statement (ii) is a corollary of statement (i)
(B) Both statements (i) and (ii) are correct
(C) Only statement (i) is correct
(D) Only statement (ii) is correct

16. Let $X_1,\ldots,X_n$ be *i.i.d.* Binomial$(1,\theta)$, where $0<\theta<1$. Then a complete sufficient statistic for θ is:

(A) $\sum_{i=1}^{n} x_i^2$

(B) $\sum_{i=1}^{n} x_i$

(C) $\bar{x}$

(D) None

17. Let X be a random variable with pmf $P(X=x)=1/N$, for $x=1,2,\ldots,N$ such that $N\geq 1$ and $P(X=x)=0$ elsewhere. A complete sufficient statistic for N is:

(A) $x_{(1)}$

(B) $x_{(n)}$

(C) $\sum_{i=1}^{n} x_i$

(D) $\sum_{i=1}^{n} x_i^2$

18. Minimal sufficient statistic for a random sample of size n drawn from a Poisson(λ) distribution is:

(A) $\sum_{i=1}^{n} x_i$

(B) $\sum_{i=1}^{n} x_i^2$

(C) $X_{(1)}$

(D) None

19. Let $e = [\text{Var}(T_1) / \text{Var}(T_2)]$ and T_1 be an MVUE and T_2 be any other estimate with variance $\frac{\sigma^2}{e}$, for a given parameter. Then, the correlation coefficient between T_1 and T_2 is:

(A) $\frac{e}{2}$

(B) $e^{1/2}$

(C) $e^{-1/2}$

(D) $2e^{1/2}$

20. Let $X_1, \ldots, X_n$ be a random sample from $U[0, \theta]$, $\theta > 0$. UMVUE of θ is:

(A) $\frac{n+1}{n} X_{(n)}$

(B) $\frac{n}{n+2} X_{(n)}$

(C) $\frac{n}{n+1} X_{(1)}$

(D) $(n+1) X_{(1)}$

21. Let X follow an exponential distribution with parameter θ such that $f(x_i) = \theta e^{-\theta x_i}$, $x_i \geq 0$. Then UMVUE of $\frac{1}{\theta^2}$ is:

(A) $\frac{(\Sigma x_i)^2}{n(n-1)^2}$

(B) $\frac{(\Sigma x_i)^2}{n^2(n-1)}$

(C) $\frac{(\Sigma x_i)^2}{n(n-1)}$

(D) $\frac{\Sigma(x_i^2)}{n(n-1)}$

22. If $X \sim \text{Gamma}(\alpha, \beta)$ for $0 \leq x < \infty$, with α known, then Var(MLE(β)) is:

(A) $\frac{n-1}{n} (\bar{x} / \alpha^2)$

(B) $\frac{1}{n(n-1)} (\bar{x} / \alpha)$

(C) $\frac{1}{n} (\bar{x} / \alpha)^2$

(D) $\frac{\bar{x}^2}{n \alpha^3}$

23. Consider a random sample from $f(x;\theta) = \begin{cases} \theta\, x^{\theta-1}, 0 \le x \le 1, \theta > 0; \\ 0\,; \text{ otherwise.} \end{cases}$

then $(1-\alpha)$ level confidence interval for θ is given as :

(A) $\frac{-\chi^2(\alpha/2),2n}{2\Sigma \log x_i}, \frac{\chi^2(1-\alpha/2),2n}{2\Sigma \log x_i}$

(B) $\frac{2\Sigma x}{\chi^2(\alpha/2),2n}, \frac{2\Sigma x}{\chi^2(1-\alpha/2),2n}$

(C) $(X_{(1)} - \log(1/\sqrt{\alpha}), X_{(1)})$

(D) None of above

24. Given the frequency function

$$f(x,\theta) = \frac{1}{\theta}\ ,\ 0 \le x \le \theta$$

For testing $H_0: \theta = 1$ against $H_1: \theta = 2$ based on a single observation x, the size of type I error for the interval $x \ge 0.5$ as critical region is

(A) $\theta - 0.5$

(B) $\theta - 1.5$

(C) $\theta + 1.5$

(D) None

25. Which of the following statements are correct?

(i) Sign test is nonparametric equivalent of t-test.

(ii) Chi- square goodness of fit requires nominal data only.

(A) Only (i)

(B) Only (ii)

(C) Both (i) & (ii)

(D) None

26. Which of the following statements are incorrect?

(i) Chi-square test can be used for both discrete and continuous distributions but K-S test cannot be used for both.

(ii) For applying Kolmogorov – Smirnov (K- S) test, the distribution should be completely specified.

(A) Only (i)

(B) Only (ii)

(C) Both (i) & (ii)

(D) None

27. A Poisson distribution has a double mode at X=1 and X=2. P(X=1) is

(A) $\frac{2}{e^2}$

(B) $\frac{4}{e^2}$

(C) $\frac{2}{e}$

(D) Cannot be determined

28. A couple decides to continue having children until they have a male child. If prevalence of a male child in their community is $\frac{1}{3}$ then the expected number of children before first male child is born is

(A) 1
(B) 2
(C) 3
(D) 4

29. Which of the following statements are incorrect?

(i) If the independent increments follow Poisson law, then the process is called Poisson process.
(ii) If the independent increments follow Gaussian law, then the process is called as Gaussian process.

(A) Only (i)
(B) Only (ii)
(C) Both (i) &(ii)
(D) None

30. Which of the following processes is a wide- sense stationary:

(i) $X(t) = Acos(w_\theta t + \theta)$,where A and w_θ are constants and θ is uniformly distributed random variable in ($0,2\pi$).

(ii) $X(t) = Acos\lambda t + Bsin\lambda t$, where A and B are random variables such that E(A)= E(B)=0, $E(A^2) = E(B^2) = k$, where k is a constant and E(AB) = 0.

(A) Only (i)

(B) Only (ii)

(C) Both (i) & (ii)

(D) None

31. Which of the following is a true fact?

(i) ANOVA can deal with two or more independent variables simultaneously
(ii) If most of the total variance is due to error, in an ANOVA , then F-ratio will be very high.

(A) Only (i)

(B) Only (ii)

(C) Both (i) & (ii)

(D) None

32. Which of the following statistical procedure deals with reducing dimensionality of data:

(A) ANOVA

(B) ANACOVA

(C) Both (a) & (b)

(D) None

33. Which of the following is true in respect of Hotelling's T^2 :

(i) It takes into account the covariance structure of data.
(ii) It is multivariate generalization of the student's t-test.

(A) Only (i)
(B) Only (ii)
(C) Both (i) & (ii)
(D) None

34. Which of the following central tendency measures is useful in averaging ratios and percentages:

(A) Arithmetic Mean
(B) Geometric Mean
(C) Mode
(D) None

35. The standard deviation of first 10 natural numbers is

(A) 1.2
(B) 2.9
(C) 4.5
(D) 5

36. Sum of Squares of deviations of all the observations is minimum when deviations are taken From

(A) Arithmetic Mean
(B) Median
(C) Mode
(D) Geometric Mean

37. In a moderately symmetrical distribution, mode and mean are given to be 32.1 and 35.4 respectively. Its median is approximately

(A) 39.1
(B) 34.3
(C) 32.9
(D) 35.2

38. Which of the following statements are incorrect ?

(i) Ogives can provide values of deciles.
(ii) Histogram are two-dimensional.
(iii) Mode of a distribution cannot be less than it's arithmetic mean
(iv) Standard deviation is independent of change of origin but not of change of scale.

(A) (i) & (ii)

(B) (ii)

(C) (iii)

(D) (iv)

39. Which of the following are not influenced by the size of extreme values in a dataset:

(i) Arithmetic Mean
(ii) Median

(A) Only (i)
(B) (ii)
(C) Both (i) & (ii)
(D) None

40. Which of the following statements is/are true about Coefficient of Correlation:

(i) It is always greater than its corresponding regression coefficients.
(ii) It is larger than its corresponding coefficient of determination except at its limiting values.
(iii) It is not symmetric in case of moderately skew data.

(A) (i)
(B) (ii) & (iii)
(C) Only (ii)
(D) Only (iii)

41. Laspeyres Index does not satisfy

(i) Circular Test
(ii) Time Reversal Test

(A) Only (i)
(B) Only (ii)
(C) Both (i) & (ii)
(D) None

42. In a $\bar{X}$ – chart, a trend of seven or more point but within control limits is indicative of

(A) increase in process variability
(B) systematic differences within subgroups
(C) gradual changing of the process level
(D) frequent adjustments in production process

43. Which of the following statements are true ?

(i) Sample allocation under proportional allocation depends on the size and heterogeneity of Strata.
(ii) Optimum allocation is a particular case of Neyman allocation.

(A) Only (i)

(B) Only (ii)

(C) Both (i) & (ii)

(D) None

44. Sampling weights under equal probability sampling within stratum, for proportional allocation of 30 sample units to 30 strata of sizes 40,60 and 50 are respectively

(A) 5
(B) 10
(C) 15
(D) 20

45. A small town has 60 hospitals and 350 beds for patient's treatment of a disease X. Using Probability Proportional to Size with replacement (PPSWR) sampling, following seven hospital are selected with data given as,

Sampled Hospital	No. Of beds	Disease X patients admitted
1	03	02
2	15	12
3	07	05
4	05	03
5	15	12
6	20	15
7	10	08

Estimate of total number of patients infected by disease X who are hospitalized in the whole town is

(A) 447
(B) 347
(C) 257
(D) 157

46. Which of the following statements are incorrect ?

(i) Two stage sampling, in general, is more efficient than cluster sampling.
(ii)Two stage sampling is less efficient than SRSWOR for positive value of intra-class correlation coefficient.

(A) Only (i)
(B) Only (ii)
(C) Both (i) & (ii)
(D) None

47. Which of the following statements about Ratio estimator is correct ?

(i) Bias of Ratio estimator is zero when the relationship between study variable and auxiliary variable is linear but does not pass through origin.

(iii) Ratio estimator will always be more efficient than the corresponding estimator under SRSWOR

(A) Only (i)
(B) Only (ii)
(C) Both (i) & (ii)
(D) None

48. The number of persons dying at age 75 is 476 and complete expectation of life at ages 75 and 76 years are 3.92 and 3.66 years respectively. Number of persons living at ages 75 and 76 respectively are

(A) 2509 ; 2478
(B) 2605 ; 2108
(C) 2675 ; 2199
(D) 2685 ; 2233

49. Which of the following statement is correct?

(A) Stable population supports Malthus Law.

(B) Overall birth and death rates are always equal under stable population.

(C) Population is closed to emigration and immigration under stationary population.

(D) Number of living persons between ages (x) and (x+1), in each year, will always remain same under stable population.

50. The demand curve and the supply curve of a commodity are given respectively by $D = 19-3p^2-p^2$ and $S= 5p-1$. Equilibrium price and quantity exchanged are respectively Given by

(A) 2 ; 9
(B) 3 ; 12
(C) 4 ; 16
(D) 5 ; 20

.....X.....